การหาค่าที่เหมาะสมที่สุดในการจัดตารางเวลารถไฟในระบบรางเดี่ยว

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บทคัดย่อ

การจัดตารางเวลารถไฟนั้นเป็นสิ่งสำคัญมากอย่างหนึ่ง ในการที่จะจัดให้รถไฟเคลื่อนที่จากต้นทางไปยังจุดปลายทาง บนระบบรางอันซับซ้อน โดยให้เกิดความล่าช้าน้อยที่สุด แต่โดยส่วนมากแล้วปัญหาของความล่าช้าจะเกิดจากปัญหาคอขวด ซึ่งจะสามารถพบได้บ่อยในระบบรางประเภทรางเดี่ยว ในบทความนี้จึงได้สร้างแบบจำลองทางคณิตศาสตร์ เพื่อที่จะหาค่าที่ เหมาะสมที่สุดในการจัดตารางเวลารถไฟในระบบรางเดี่ยว แบบจำลองทางคณิตศาสตร์ที่สร้างขึ้นมานั้นมีเป้าหมายเพื่อที่จะหา ระยะเวลาเดินทางที่น้อยที่สุด ในขณะที่คำตอบสามารถทำให้สมการเงื่อนไขเป็นจริงได้ทุกประการ เช่น รถไฟจะต้องเคลื่อนที่ อย่างต่อเนื่อง รถไฟจะต้องไม่เกิดการสวนกันระหว่างรางเดี่ยว และ ณ ช่วงเวลาหนึ่ง บนแทร็กย่อยใดๆนั้นสามารถรองรับ รถไฟได้เพียงหนึ่งขบวนเท่านั้น เป็นต้น ผลลัพธ์ที่ได้มาจากแบบจำลองทางคณิตศาสตร์มานั้นจะแสดงในรูปแบบของ ตารางเวลาที่รถไฟทุกขบวนจะเข้าสู่แต่ละสถานี และ ผลลัพธ์นั้นจะถูกสร้างเป็นแผนภาพ โดยการพล๊อตระหว่างระยะทางและ เวลา เพื่อที่จะได้เห็นภาพได้อย่างชัดเจนยิ่งขึ้น วิธีนี้ไม่เพียงแต่จะคำนวณคำตอบออกมาในเวลาที่สามารถยอมรับได้ แต่จะ รวดเร็วกว่าวิธีเดิมเป็นอย่างมาก และ ค่าตารางเวลาที่ได้จากการหาค่าที่เหมาะสมที่สุดจะสามารถทำให้กระบวนการจัด ตารางเวลารถไฟมีประสิทธิภาพมากขึ้นอีกด้วย

คำสำคัญ: การจัดตารางเวลา, รถไฟ, แบบจำลองทางคณิตศาสตร์

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Optimization of Train Scheduling in a Single Rail Network

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Abstract

Train scheduling in a rail network is very important for controlling a hundred of trains moving through a complex network. Mostly, the delay is caused by bottle neck problems, which frequently occur in a single rail network. As a result, a mathematical model was applied to optimize the train scheduling in a single rail network with flexible path. The objective is to minimize travelling time of all trains while satisfying all operational constraints, For example, the trains must keep moving continuously, train confliction should be avoided for safety reason and a track contains only one train at a time. A solution was achieved by having travelling time of all trains at all stations including arrival and departure time of the trains reported and reflected in form of graph by plotting data between distance and time until no conflict occurs in the system, after all. This method not only can compute an acceptable time but also is quicker than the original way. An optimal time table solution was obtained through optimization, which made scheduling process much more efficiently.

Keywords: Scheduling, Train, Mathematical Model

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1. Introduction

Rail transportation becomes more significant in a lot of countries. Railway industry requires huge investments in infrastructure, vehicles and crew employees. Few years ago, train scheduling was one of the important issues in rail transport management to control a hundred of trains in a complex network; the train scheduling timetable should be developed. The train scheduling is only small element of rail management that does not need a lot of funds. However, only small percentage increasing in efficiency may bring about huge benefits itself.

Researches on train scheduling problems have been increased over a decade. In author's review, [1] is a researcher, who developed a model of train scheduling problems in form of disjunctive graph in a single rail network which aimed to minimize, and solved delays by exact method. In addition, mathematical models, with same objectives, functions and other constraints for train scheduling problems, have been developed by many researchers [2-4]. Other researcher also studied on train scheduling problems in a single rail network by using branch and bound method to reduce space searching. According to his model, the system contained 14 platforms and 31 trains [5].

Few years later, [6] developed a model of train scheduling problems by adding no-wait constraint. He also assert that "Blocking constraint cannot be modeled in form of disjunctive graph". Thus, a problem was modeled into an alternative graph which was developed by [7]. [8] studied on bottle neck area and modeled a train scheduling problem in form of job-shop scheduling with no-wait constraint. He also used branch and bound method similar to [5]. An idea to model the train scheduling problems into blocking parallel machine job-shop scheduling was presented by [9]. Besides a Hueristic name FSP (feasibility satisfaction procedure) was created to solve the problem above. [10-11] focused on development

of train scheduling problems and also modeled in form of alternative graph. Mostly, the train scheduling problem was modeled through job-shop scheduling problem in form of alternative graph model by defining trains as job, and tracks as machine. In addition many researchers were used no-wait and blocking –parrallel constraints to solve the conflict problems.

While most researchers studied on a whole train scheduling optimization, a few researchers focused on train dispatching mechanism. [12], who developed Greedy TAS method for selecting trains into a platform when more than one train needed to use the same platform at the same time. [13] presented a switchable dispatching policy for multiple tracks, which selected faster trains to pass before slower trains. A dispatching issue in train scheduling problems is also very important and difficult to be solved.

Generally, the model of train scheduling problem aims to minimize the travelling time that means total delay is decreased by its self. A crisis which that causes a huge delay in the train scheduling is the bottle neck problems, which frequently occur in a single rail network. This train scheduling problem is therefore taken into consideration in a single rail network with flexible path and fixed train velocity, which is modeled through mathematical model and solved by exact method. The objective is also aim to minimize the total travelling time of all trains, while all operation constraints are met. To develop a better train schedule, it may lead to an increase of rail utilization rate and likely to make more profits.

2. Train Scheduling Problem

A train scheduling problem is similar to job-shop problem by determining a train as job flow from origin to destination, and a track as a machine that can process only one job at a time, and a train running time on any track as processing time of job on any machine. Base on

earlier researches, most researchers modeled a train scheduling as job shop scheduling problem, due to its complex and large space searching problem, also known as NP-Hard problem. Thus, this mathematical model tried to model train scheduling problem into linear problem by reducing a real complex network. A single rail network was inspired by a local northern trail. The rail networks were divided into small parts by "node", which represented a track number and connected to other nodes by "links", which referred to track distance as shown in Fig. 1. All trains have moved through these links from origin to destination while visiting all stations, the links were divided into sub-links for waiting and shunting.
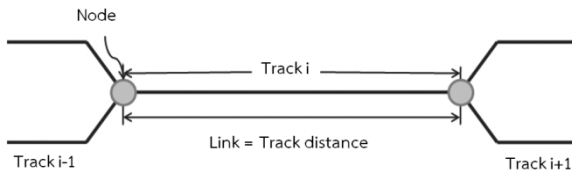

Fig. 1 Simple Rail Network Model

Certainly, one sub-link can contain only one train at a time. In addition, to assure safety condition, a headway time will be used to separate two trains from each other. All train priority is equal. In train scheduling problem, arrival and departure time of all trains will be calculated to build an optimal train timetable. Finally, in this model, train acceleration and deceleration time are negligible.

Mathematical model for train scheduling problem:

Notation an index number

i Trains index number

j Track index number

k Sub-track index number

L A set of train

Q A set of track

EB Set of train moving from east to west

WB Set of train moving from west to east

Q_i Set of track occupied by train i

K_j A set of sub-tracks on track j

D_{ijk} Dwell time for train i on sub-tracks k^{th} of track j

M A very large number

c_j Track j limit velocity

d_{jk} Distance of track j on sub-track k^{th}

v_i Train velocity

$w_{ijk} = \begin{cases} 1 \text{ if train i stop at k}^{th} \text{ sub-track on track j} \\ 0 \text{ otherwise} \end{cases}$

Decision variables

T_{ijk}^{inc} Arrival time of train i on sub-track k^{th} of track j

T_{ijk}^{out} Departure time of train i on sub-track k^{th} of track j

$H_{ijk} = \begin{cases} 1 \text{ if train i used k}^{th} \text{ sub-track on track j} \\ 0 \text{ otherwise} \end{cases}$

$A_{irjk} = \begin{cases} 1 \text{ if select train i into track before train r} \\ 0 \text{ otherwise} \end{cases}$

$B_{irjk} = \begin{cases} 1 \text{ if select train i into track before train r} \\ 0 \text{ otherwise} \end{cases}$

$C_{irjk} = \begin{cases} 1 \text{ if select train i into track before train r} \\ 0 \text{ otherwise} \end{cases}$

Travelling time for each train moving on any sub-track can be computed by equation (1)

$$T_{ijk} = T_{ijk}^{out} - T_{ijk}^{inc} \qquad (1)$$

Therefore an objective function can be calculated by summation of T_{ijk}, as shown in equation (2)

$$\min \sum_{i=1}^{L} \sum_{j \in Q_i} \sum_{k \in K_j} T_{ijk} \qquad (2)$$

Subject to;

$$T_{ijk}^{out} \geq T_{ijk}^{inc} + (H_{ijk} * \frac{d_{jk}}{v_i}) + D_{ijk} * w_{ijk}$$
$$\forall i \in EB, j, k \qquad (3)$$

$$T_{ijk}^{out} \geq T_{ijk}^{inc} - (H_{ijk} * \frac{d_{jk}}{v_i}) - D_{ijk} * w_{ijk}$$
$$\forall i \in WB, j, k \qquad (4)$$

$$T_{ijk}^{out} - T_{ijk}^{inc} \geq (H_{ijk} * \frac{d_{jk}}{c_j}) \tag{5}$$

$$\sum_{k \in K_j} H_{ijk} = 1 \quad \forall i, j \tag{6}$$

$$H_{ijk} * M \geq T_{ijk}^{inc} \quad \forall i, j, k \tag{7}$$

$$H_{ijk} * M \geq T_{ijk}^{out} \quad \forall i, j, k \tag{8}$$

$$\sum_{k \in K_j} T_{ijk}^{out} = \sum_{k \in K_j} T_{i,j+1,k}^{inc} \quad \forall i, 1 \leq j \leq Q_i \tag{9}$$

$$T_{ijk}^{out} \leq T_{rjk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - A_{irjk})]$$
$$\forall i \in EB, k \in EB, j \in Q_i \tag{10}$$

$$T_{rjk}^{out} \leq T_{ijk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - A_{rijk})]$$
$$\forall i \in EB, k \in EB, j \in Q_i \tag{11}$$

$$T_{ijk}^{out} \leq T_{rjk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - B_{irjk})]$$
$$\forall i \in WB, k \in WB, j \in Q_i \tag{12}$$

$$T_{rjk}^{out} \leq T_{ijk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - B_{rijk})]$$
$$\forall i \in WB, k \in WB, j \in Q_i \tag{13}$$

$$T_{ijk}^{out} \leq T_{rjk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - C_{irjk})]$$
$$\forall i \in WB, k \in EB, j \in Q_i \tag{14}$$

$$T_{rjk}^{out} \leq T_{ijk}^{inc} + M[(1 - H_{ijk}) + (1 - H_{rjk}) + (1 - C_{rijk})]$$
$$\forall i \in WB, k \in EB, j \in Q_i \tag{15}$$

Constraints (3) and (4) ensure that departure time must be greater than arrival time plus dwelling time and running time, in equation (3) calculate travelling time only east bound train, in the same way equation (4) calculate travelling time only west bound train. A running time in any track is computed from track distance divided by train velocity. Constraint (5) ensures that the train velocity cannot exceed a track limit velocity.

Constraint (6) ensures that any sub-link can contain only one train at a time. Constraints (7) and (8) force that arrival and departure time must be greater than zero when binary variable H is more than zero. To keep the trains moving continuously, departure time must be equal to arrival time when a train enters the next track as shown in equation (9). Equations (10-11) represent a conflict free constraint for both east bound trains. Equations (12-13) represent a conflict free constraint for both west bound trains. Equations (14-15) represent a conflict free constraint for west bound train, by giving headway time to each train and holding on until the next track is available.

3. Numerical example

In this section, a numerical example was presented for model testing purpose. All fixed data in this model were assumed.

In the example, a small local network was considered as shown in Fig. 2. A railroad network contained 15 tracks consisting of 11 real tracks and 4 dummy tracks referring to 4 platforms. Numbers of the dummy tracks were not given. They were used for dwelling time at each station. A dwelling time value was shown in Table 1. The system included 3 trains travelling from east to west, entering the system via track 1 and leaving the system at track 11. In the same way, 2 west bound trains moving in opposite direction at the same time. West bound train entered the system at track 11 and leaving from track 1. A rail network consisted of 11 tracks, each of which contained different numbers of sub-tracks, such as track 2 had 4 sub-tracks, i.e. track 2 capable of holding 4 trains at the same time at its maximum capacity.
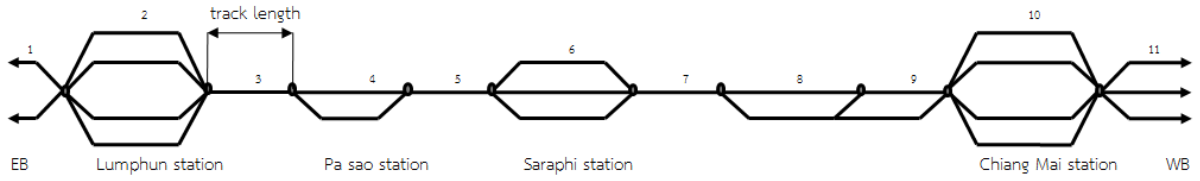
Fig. 2 a simple railway network

All trains can move on any sub-track, that is, the trains entering track 1 sub-track 1 can enter track 2 on any sub-tracks, except track 8 sub-track 1 unable to enter track 9 sub-track 2 by east bound train. In addition, a platform were placed into the system on tracks 2, 4, 6 and 10, where the trains must stop for dwelling in. Parameter variables were used in this model, velocity of all trains was set at 40-50 Km/h as shown in Table 1. Running time for each train can be calculated by dividing distance of each track by velocity. In each track, velocity of all trains was limited, i.e. the trains are unable to run with limited velocity allowed for the track. For example, though a train can run at 40 km/h, but if its maximum velocity of the current track is at 35 km/h, the maximum speed that the train is allowed to run will be at 35 km/h.

Firstly, mathematical models were applied to LINGO software for finding the optimal train timetable with flexible path. In LINGO software, the dummy tracks at each

platform placed into tracks 2, 4, 6 and 10 were added to keep the train dwelling time increasing while distances were fixed. For easy way to find where conflict occurred in the system, LINGO was set to import data from Excel, and export data back to Excel for plotting solution in form of graph as shown in Fig.3 and Fig.4. Due to a lot of decision variables (around 900 decision variables), only some examples of decision variables of train 1 and train 4 were given as shown below.

Total travelling time for both train 1 and train 4 was 41.85 min. along the distance of 23

kilometers and the whole train is 213.72 minute. For better understanding, this was presented in form of graph as shown in Fig 4 and Fig 5.

East bound: Trains 1 Decision variables

(1) Binary variable, $H_{1,1,1} = H_{1,2,2} = H_{1,3,1} = H_{1,4,2} = H_{1,5,1} = H_{1,6,3} = H_{1,7,1} = H_{1,8,2} = H_{1,9,3} = H_{1,10,2} = H_{1,11,3} = 1$

(2) Arrival time, $T_{1,1,1}^{inc} = 14.43$, $T_{1,2,2}^{inc} = 16.65$, $T_{1,3,1}^{inc} = 19.15$, $T_{1,4,2}^{inc} = 28.66$, $T_{1,5,1}^{inc} = 30.66$, $T_{1,6,3}^{inc} = 43.07$, $T_{1,7,1}^{inc} = 44.57$, $T_{1,8,2}^{inc} = 55.08$, $T_{1,9,3}^{inc} = 55.84$, $T_{1,10,2}^{inc} = 57.12$, $T_{1,11,3}^{inc} = 59.12$

(3) Departure time, $T_{1,1,1}^{out} = 15.45$, $T_{1,2,2}^{out} = 19.15$, $T_{1,3,1}^{out} = 27.76$, $T_{1,4,2}^{out} = 30.66$, $T_{1,5,1}^{out} = 42.07$, $T_{1,6,3}^{out} = 44.57$, $T_{1,7,1}^{out} = 55.08$, $T_{1,8,2}^{out} = 55.84$, $T_{1,9,3}^{out} = 56.52$, $T_{1,10,2}^{out} = 59.12$, $T_{1,11,3}^{out} = 59.32$

West bound: Trains 4 Decision variables

(1) Binary variable, $H_{4,1,2} = H_{4,2,4} = H_{4,3,1} = H_{4,4,1} = H_{4,5,1} = H_{4,6,1} = H_{4,7,1} = H_{4,8,2} = H_{4,9,3} = H_{4,10,2} = H_{4,11,3} = 1$

(2) Arrival time, $T_{4,1,2}^{inc} = 43.87$, $T_{4,2,4}^{inc} = 40.17$, $T_{4,3,1}^{inc} = 31.56$, $T_{4,4,1}^{inc} = 28.66$, $T_{4,5,1}^{inc} = 17.25$, $T_{4,6,1}^{inc} = 14.75$, $T_{4,7,1}^{inc} = 4.24$, $T_{4,8,2}^{inc} = 3.48$, $T_{4,9,3}^{inc} = 2.8$, $T_{4,10,2}^{inc} = 0.2$, $T_{4,11,3}^{inc} = 0$

(3) Departure time, $T_{4,1,2}^{out} = 44.89$, $T_{4,2,4}^{out} = 42.67$, $T_{4,3,1}^{out} = 40.17$, $T_{4,4,1}^{out} = 30.66$, $T_{4,5,1}^{out} = 28.66$, $T_{4,6,1}^{out} = 16.25$, $T_{4,7,1}^{out} = 14.75$, $T_{4,8,2}^{out} = 4.24$, $T_{4,9,3}^{out} = 3.48$, $T_{4,10,2}^{out} = 2.2$, $T_{4,11,3}^{out} = 0.2$

4. Conclusion

This paper presented the other mathematical models to deal with the train scheduling problems in a single rail network with flexible path. A mathematical model was set as LP to guarantee a global optimum. For easy way to find where conflict occurred in the system, LINGO software was set to import data from Excel. The problem was solved through a mathematical model by LINGO software and the data was exported back to Excel for plotting solution in form of graph. The objective of this model was to minimize the travelling time of all trains while satisfying all constraints.

According to the example, the model was tested by considering a small local network. A limit velocity of all trains in each track was determined. The system included 3 trains travelling from east to west.

Table 1 parameter value

Parameter	Value	Parameter	Value
d_1	500	v_5	50
d_2	600	c_1	30
d_3	5020	c_2	30
d_4	450	c_3	35
d_5	7000	c_4	30
d_6	500	c_5	50
d_7	7000	c_6	30
d_8	500	c_7	50
d_9	450	c_8	50
d_{10}	300	c_9	50
d_{11}	100	c_{10}	30
M	9999	c_{11}	30
v_1	40	D_{LP}	2

Parameter	Value	Parameter	Value
v_2	45	D_{PS}	2.5
v_3	40	D_{SRP}	1.5
v_4	40	D_{CM}	2

Likewise, 2 west bound trains moved in opposite direction at the same period. A railroad network contained 15 tracks consisting of 11 real tracks and 4 dummy tracks referring to 4 platforms.

The solution reported the travelling time of all trains at all stations included arrival and departure time and reflected in form of graph by plotting data between distance and time. As a result, no conflict was found in the system, after all.

This method can be computed in an acceptable time and also quicker than original way. An optimal time table solution was obtained through optimization, which made scheduling process much more efficiently.

This model is a small element of the main study, which focused on scheduling and timetabling but not consider in cost issues. In primary study, a short rail network data was collected from Chiang Mai Province to Lumphun Province for test a mathematical model. In the future, the expansion of this model will use the real northern trail network to study that include with multiple tracks and more complex networks and if these can fill with more data, it may lead to completely solutions.

Future work, this model can be dealt with a real single rail scheduling problem by considering train priority and speed limit on each track, and also needs to add no-wait - blocking and other constraints.

Departure Time	East bound			West bound		Arrival Time		East bound			West bound	
(min)	Train1	Train2	Train3	Train4	Train5	Cumulative Distance		Train1	Train2	Train3	Train4	Train5
Into LM St	15.45	41.49	65.01	44.89	70.93	510	0	14.43	40.47	63.99	43.87	69.91
LM St	16.65	42.69	66.21	43.87	69.91	1110	0.1	15.45	41.49	65.01	42.67	68.71
Dwell LM	19.15	45.19	68.71	42.67	68.71	1110	0.1	16.65	42.69	66.21	40.17	66.21
Link LP-PS	27.76	53.8	77.32	40.17	66.21	6130	0.2	19.15	45.19	68.71	31.56	57.6
PS St	28.66	54.7	78.22	31.56	57.6	6580	0.3	27.76	53.8	77.32	30.66	56.7
Dwell PS	30.66	56.7	80.22	30.66	56.7	6580	0.3	28.66	54.7	78.22	28.66	54.7
Link PS-SRP	42.07	66.84	91.63	28.66	54.7	14180	0.4	30.66	56.7	80.22	17.25	45.57
SRP St	43.07	67.84	92.63	17.25	45.57	14680	0.5	42.07	66.84	91.63	16.25	44.57
Dwell SRP	44.57	69.34	94.13	16.25	44.57	14680	0.5	43.07	67.84	92.63	14.75	43.07
Link SRP-CM 1	55.08	78.68	104.64	14.75	43.07	21680	0.6	44.57	69.34	94.13	4.24	34.66
Link SRP-CM 2	55.84	79.35	105.4	4.24	34.66	22180	0.7	55.08	78.68	104.64	3.48	34.05
Link SRP-CM3	56.52	79.95	106.08	3.48	34.05	22630	0.8	55.84	79.35	105.4	2.8	33.5
CM St	57.12	80.55	106.68	2.8	33.5	22930	0.9	56.52	79.95	106.08	2.2	32.9
Dwell CM	59.12	82.55	108.68	2.2	32.9	22930	0.9	57.12	80.55	106.68	0.2	30.9
Link Yard	59.32	82.75	108.88	0.2	30.9	23030	1	59.12	82.55	108.68	0	30.7
Travelling time	43.87	41.26	43.87	44.69	40.03	Locating Plot						
Total Travelling Time		213.72										

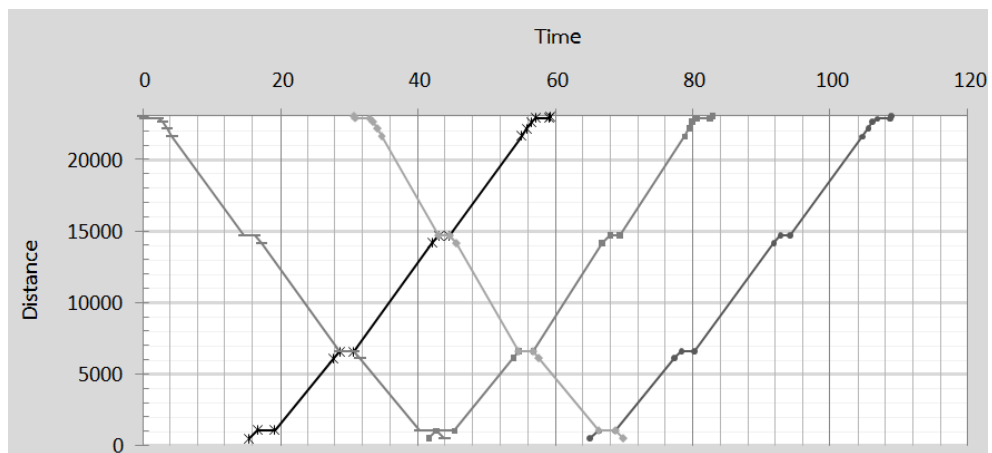Fig. 3 Arrival and Departure Timetable

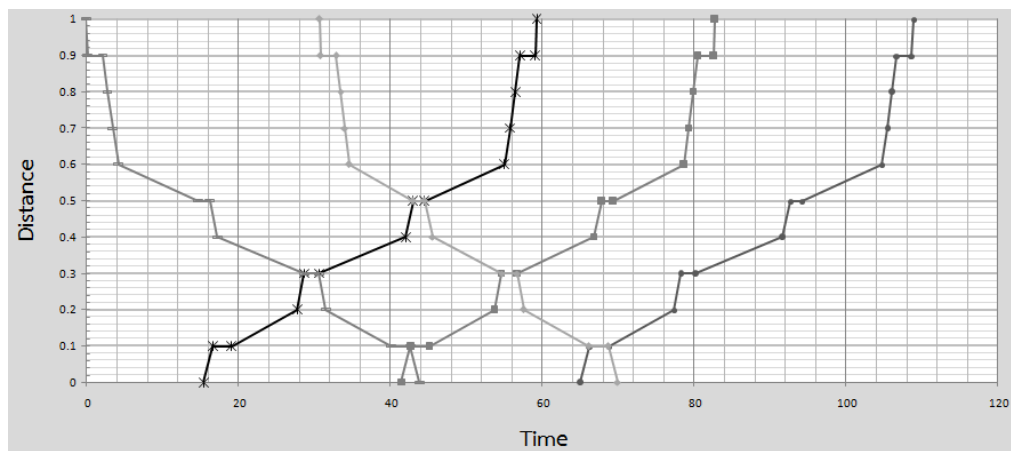Fig. 4 Train Movement Plot between Distance in Kilometers and Time Period

Fig. 5 Train Movement Plot between Distance in Located Portion and Time

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